Testing *In Situ* Aluminium Cell Control With the Dyna/Marc Cell Simulator

Marc Dupuis
GéniSim Inc.
3111 Alger St.
Jonquière, Québec, Canada G7S 2M9
marc.dupuis@genisim.com

Michael C. Schneller
In Situ Aluminium Cell Control
Robert College, Arnavutköy 34345
Istanbul, Turkey
michaelschneller@hotmail.com

ABSTRACT

Recently, a new approach to controlling alumina feeding in a H.H. cell has been proposed [1, 2 and 3]. This approach, labeled *In Situ* aluminium cell control, is based on a correlation between the slope of the cell voltage during a no feed observation period and the dissolved alumina concentration in the bath.

Once the dissolved alumina concentration has been estimated, a primary calibration surface uniquely relating the dissolved alumina concentration and the anode-cathode-distance (ACD) to the cell voltage is used to estimate the cell ACD. Once both the alumina concentration and the ACD have been estimated at the end of the observation period, a PID controller can be used to regulate the rate of alumina feeding using only the normalized cell voltage to calculate the error between the estimated dissolved alumina concentration and a target value.

The *In Situ* aluminium cell control algorithm has now been tested with the Dyna/Marc [4, 5 and 6] cell simulator demonstrating that it should be possible to control a H.H. cell in this manner with less variation in the dissolved alumina concentration and other process variables producing a prediction of improved cell current efficiency and power efficiency.

IN SITU ALUMINIUM CELL CONTROL

A new approach to controlling alumina feeding in a H.H. cell, called *In Situ* aluminium cell control has been recently proposed [1, 2 and 3]. The core principle is quite simple: it turned out that it is possible to extract the values of both the dissolved alumina concentration and the ACD at the end of a 5 to 10 minute no feed observation period by using only the standard cell amperage and voltage data.

Once this is done, it is possible for an extended period of time to use a PID controller to regulate the rate of alumina feeding using only the normalized cell voltage to calculate the error between the estimated dissolved alumina concentration and a target value. After an extended period of 8 to 12 hours, a new no feed observation period must be performed in order to reestablish the value of both process variables. The detailed procedure is described below.

Calculation of the normalized cell voltage

The first step is a calculation of the normalized cell voltage using raw sampled cell amperage and voltage data points in order to remove the effect of the line amperage fluctuations on the cell voltage fluctuations. Calculating normalized cell voltage is an alternative to calculating cell pseudo-resistance. This variable can then be used to assess the change of cell voltage/resistance due to changes in cell bath chemistry and ACD which are independent of the cell amperage.

$$V_n = (V - \text{BEMF}) / I \; * I_n + \text{BEMF} \qquad (1)$$

Where:

V_n	is the normalized cell voltage mostly free of fluctuation due to the amperage fluctuation (V)
V	is the raw fluctuating cell voltage (V)
I	is the raw fluctuating cell amperage (kA)
I_n	is the nominal or target cell amperage (kA)
BEMF	is the extrapolated voltage at zero amperage usually set to 1.65 (V)

The Dyna/Marc cell simulator can be used to illustrate the efficacy of this procedure. It can be set up to produce a noisy cell amperage but also a cell resistance without MHD and bubble release fluctuations. In this way raw cell voltage noise is only dependant on the amperage noise as shown in Figure 1.

As already demonstrated in [7], employing an incorrect value for the BEMF will lead to the calculation of a noisy cell pseudo-resistance as can be seen in Figure 2, but also of a cell pseudo-resistance having a significantly different value. As we can see in Figure 3, using an incorrect value for the BEMF will lead to the calculation of a noisy normalized cell voltage but will not change the average value that is not affected by the choice of the value of BEMF.

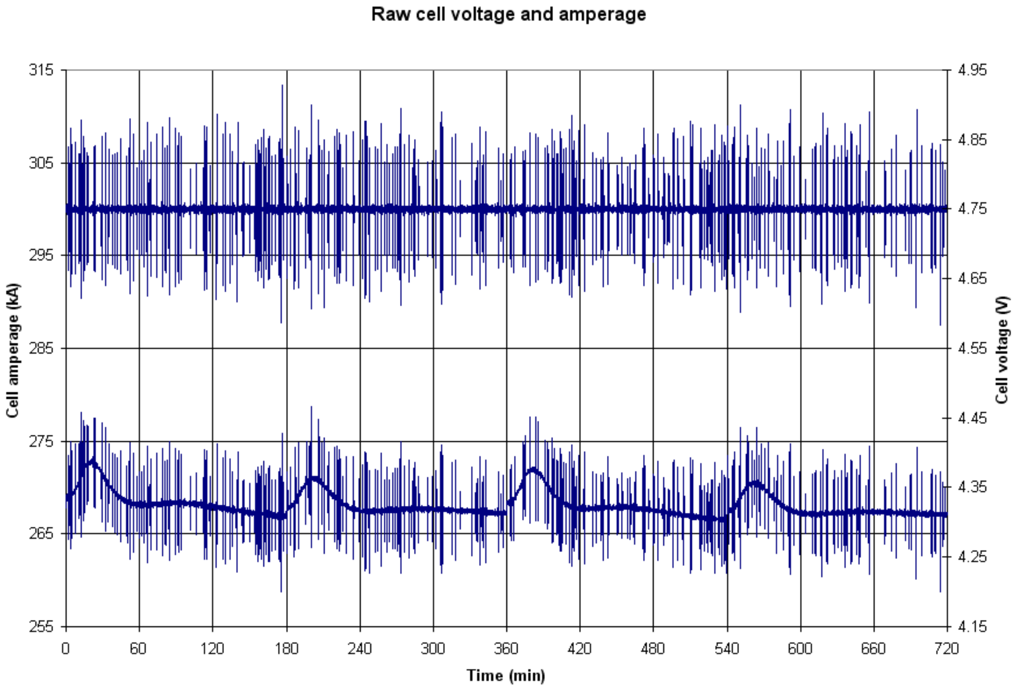


Figure 1: Raw cell voltage and amperage

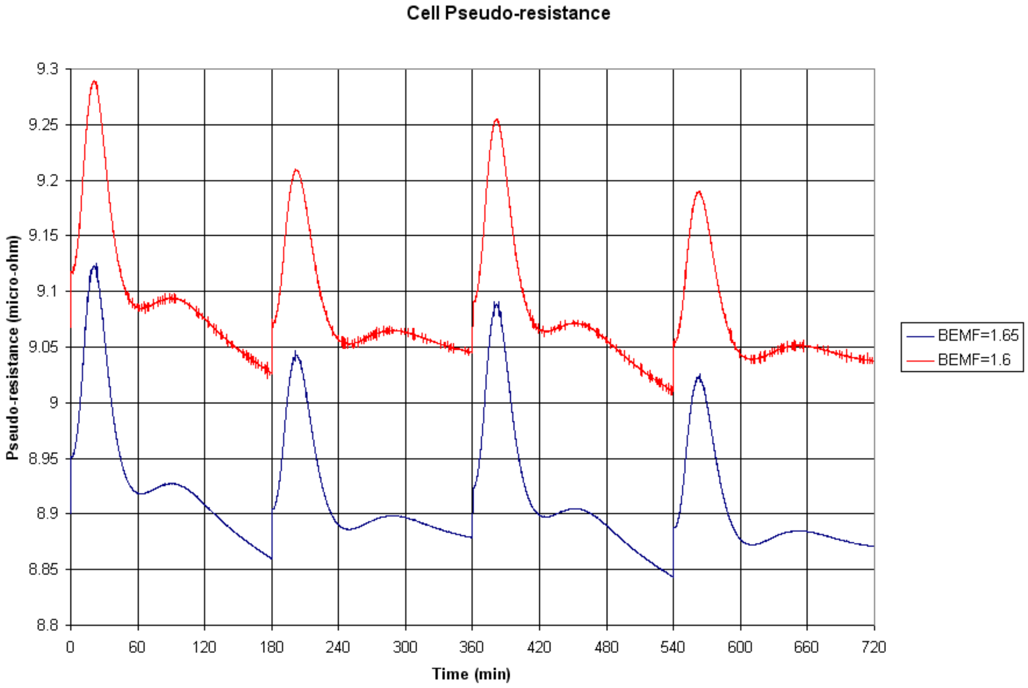


Figure 2: Cell pseudo-resistance for two BEMF values

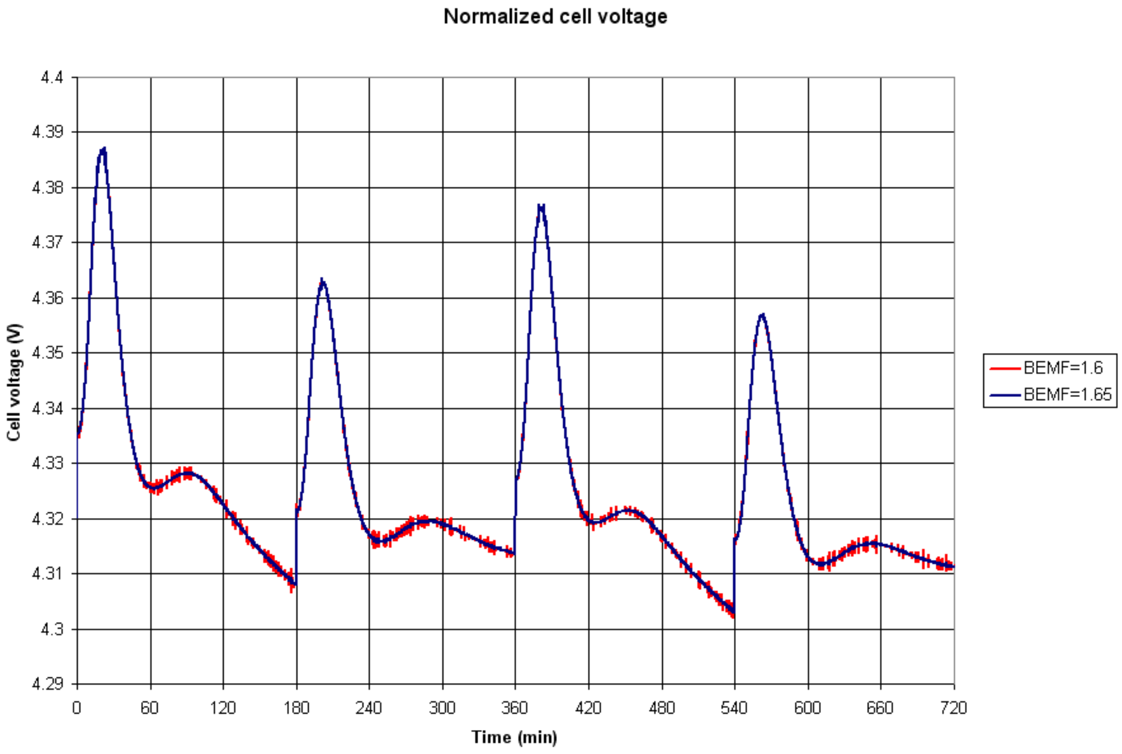


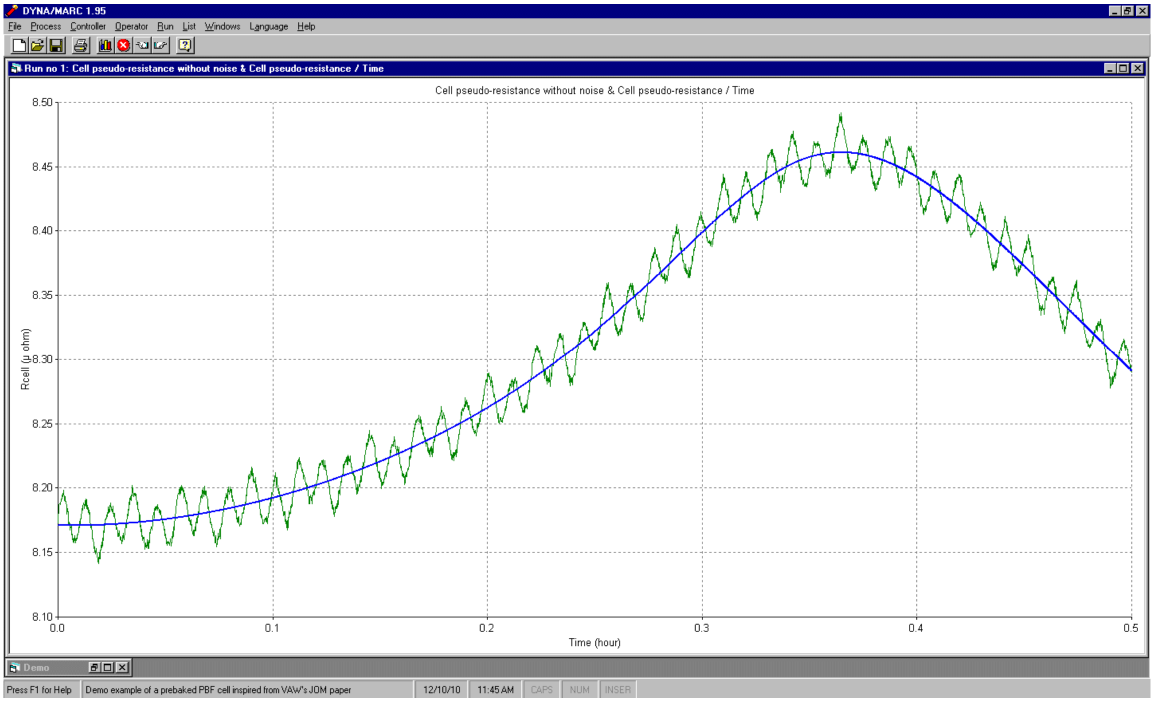
Figure 3: Normalized cell voltage for two values of the BEMF

Smoothing/fitting the normalized cell voltage

Amperage fluctuations are not the only phenomena adding higher frequency variation or noise on the raw cell voltage. Metal pad roll generated by magneto-hydro-dynamic (MHD) and CO_2 bubble gas release are two extra phenomena that add noise to the cell electrical resistance either expressed as cell pseudo-resistance or normalized cell voltage.

Dyna/Marc can be used to generate noisy cell resistance data which is accomplished by adding higher frequency ACD fluctuations in addition to the very slowly evolving ACD. Since Dyna/Marc knows the value of the slowly evolving ACD before the addition of the extra higher frequency fluctuation, it can calculate the noise free cell pseudo-resistance in addition to the noisy cell pseudo-resistance (see Figure 4).

To generate those results, Dyna/Marc was run using a 0.1 seconds time steps. The ACD noise generation subroutine available in Dyna/Marc generated a higher frequency ACD fluctuation to mimic metal roll and CO_2 gas release. That noise can be set to be proportional to ACD, mass of sludge and ledge toe position by user defined parameters. The aim of any cell controller smoothing/fitting algorithm is to generate the blue curve using data from the noisy green curve in Figure 4.



Demo example of a prebaked PBF cell inspired from VAW's JOM paper

Figure 4: Noisy and noise free evolution of the cell pseudo-resistance

The first question that comes to mind, considering the nature of a noisy signal, is what would be the appropriate data sampling rate and is the moving average calculation helpful in decreasing this type of noise?

To remove the noise having a frequency of about 0.008 Hz, it is required to perform moving average calculations over a span of at least 2 minutes, which means more than 1200 data points if a 10 Hz data sampling rate is used. As can be seen in Figure 5, the higher frequency noise is almost completely removed and the resulting red curve is a good fit of the blue curve in Figure 4.

Notice that the red curve locates the calculated average voltage at the average time which means that the last calculated value was the average voltage 1 minute ago. The black curve is the moving average curve the way Excel presents it, which is incorrect. In order to assess the averaged or smoothed value at the time of the last data point collected (which is assumed to be the present time), an extrapolation of the red curve would need to be performed which would require some kind of data fitting.

Moving averages can also be used to reduce the number of data points to perform data fit calculations. For example, collecting 10 Hz and saving only the 6 seconds moving averaged values every 6 seconds. This would remove very high frequency noise of 0.2 Hz and higher so it would remove CO_2 gas bubble release noise which is estimated to be about 1 Hz (see Figure 6).

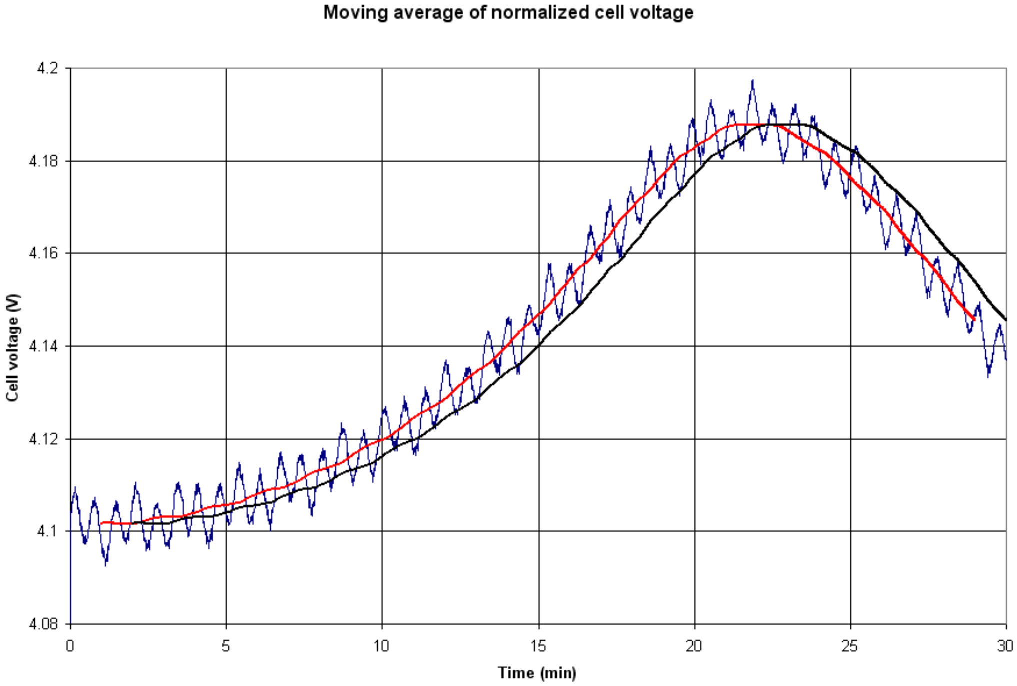


Figure 5: 2 minutes moving average of the normalized cell voltage

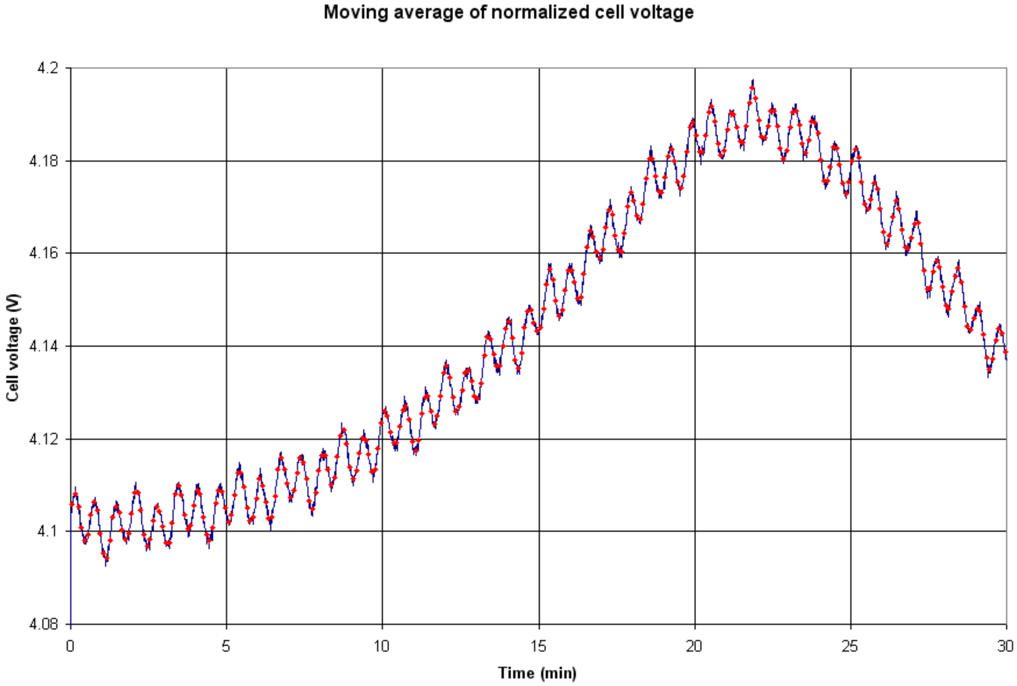


Figure 6: 6 seconds moving average of the normalized cell voltage

Assuming there is available a gas bubble release noise free 5 second moving average data point, the next step is to perform some kind of curve fitting on those points in order to eliminate the metal roll noise. The simplest approach would be to use linear root mean square (RMS) date fitting. It was already demonstrated in [8] that using linear RMS data fitting is a reliable way to estimate the slope of the normalized cell voltage (or alternatively the slope of cell pseudo-resistance). The secret of the accuracy is the selection of the time period to be fitted. It was estimated that a 5 minutes was sufficient to eliminate the metal roll noise but short enough to still be a good representation of the evolution of the noise free normalized cell voltage curve (i.e. not significant curvature). Figure 7 presents the results of such a 5 minutes fit using 60 data points.

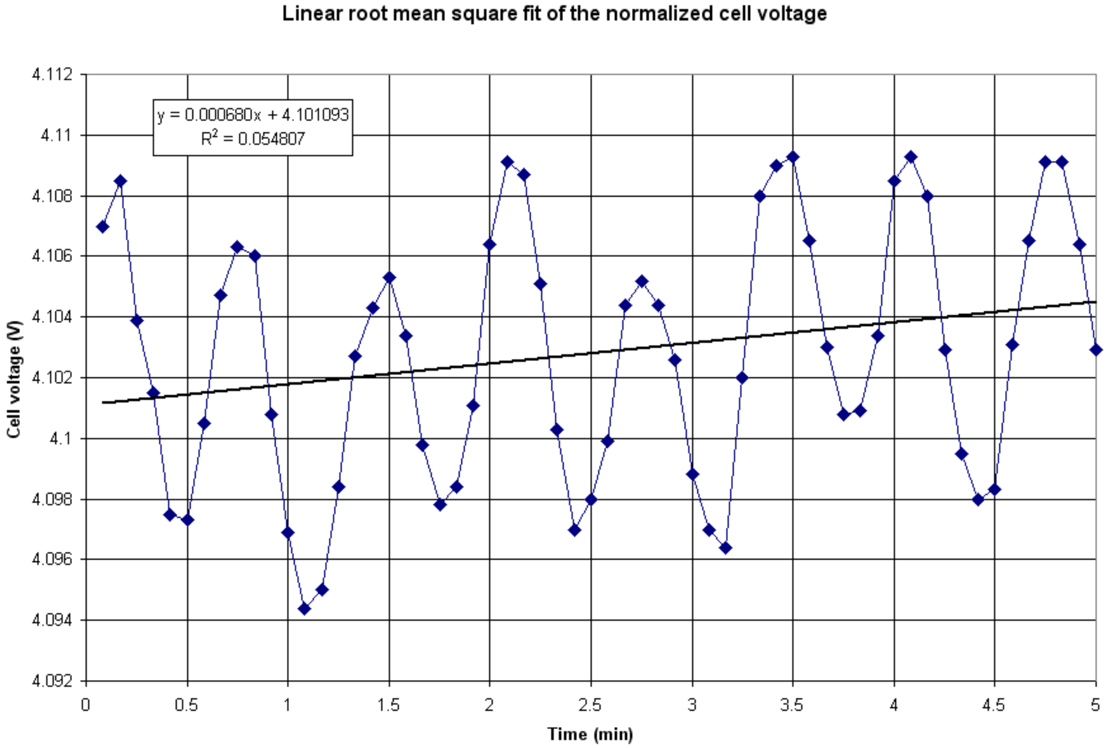


Figure 7: Linear root mean square fit of the normalized cell voltage

This fit provides two important results; first it gives a prediction of the current noise free normalized cell voltage: 0.000680 * 5 + 4.1011 = 4.1045 V.

Second it gives a prediction of the current noise free slope of the normalized cell voltage: 0.00068 V/min or 0.68 mV/min.

Repeating this data curve fitting exercise every 5 minutes leads to the successive straight line fits presented in Figure 8. It is obviously possible to repeat the calculation more often than every 5 minutes. For example this can be done every 2.5 minutes or even every minute by using the last 5 minutes of data.

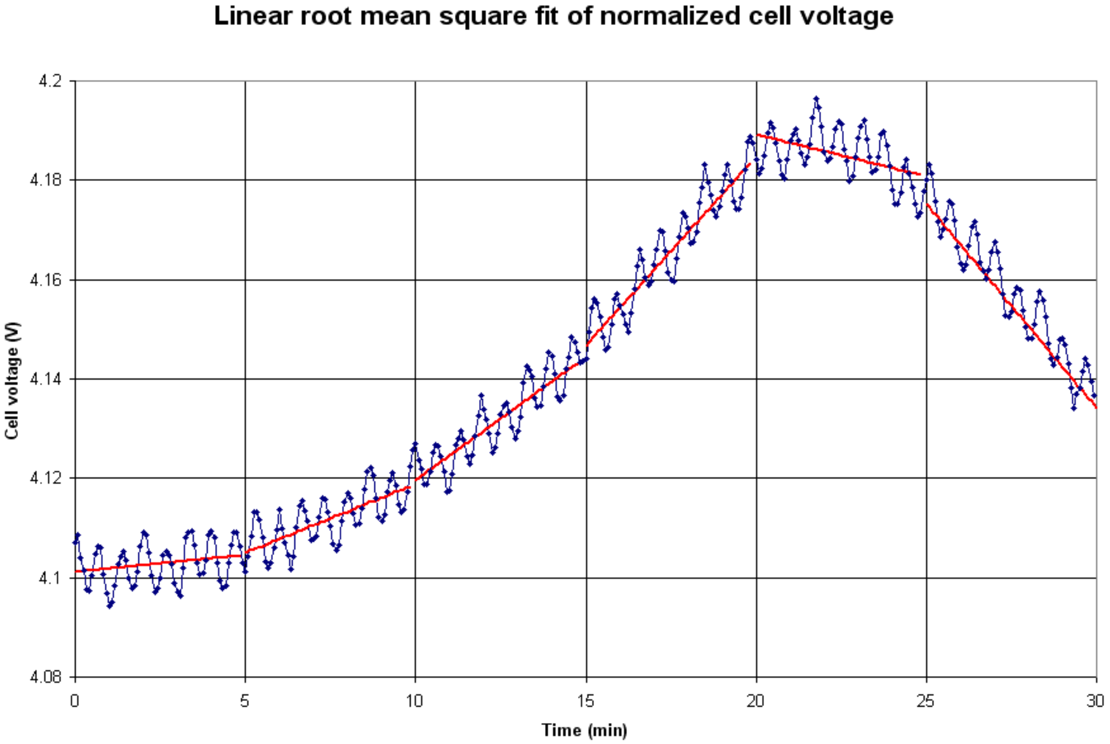


Figure 8: Linear root mean square fit of the normalized cell voltage

It is also possible to repeat a similar RMS data fit exercise using a quadratic equation instead of a linear equation. In [8] it was demonstrated that using quadratic data fitting can result in a less accurate estimation of the slope of the cell pseudo-resistance. Yet, it all depends on the choice made for the sampling frequency, the moving average calculation frequency and the data fit calculation time period.

Clearly, a longer time period is required if quadratic data fitting is employed instead of linear data filtering. When using quadratic data fitting, the curvature of the noise free normalized cell voltage must be significant enough to be well estimated. Figure 9 presents the results of a 10 minutes fit using 120 data points. Quadratic fitting of 120 data points obviously requires more CPU processing resources than a linear fitting of 60 data points.

Again, the fit provides two important results. First, it gives a prediction of the current noise free normalized cell voltage: 0.00021*100 - 0.000301*10 + 4.1019 = 4.1199 V.

Second, it gives a prediction of the current noise free slope of the normalized cell voltage: 2 * 0.00021 * 10 - 0.000301 = 0.0039 V/min or 3.9 mV/min.

Waiting 10 minutes to get a new estimate of the slope of the normalized cell voltage is certainly not a good option. Therefore, Figure 10 presents results by repeating the data fit every 5 minutes.

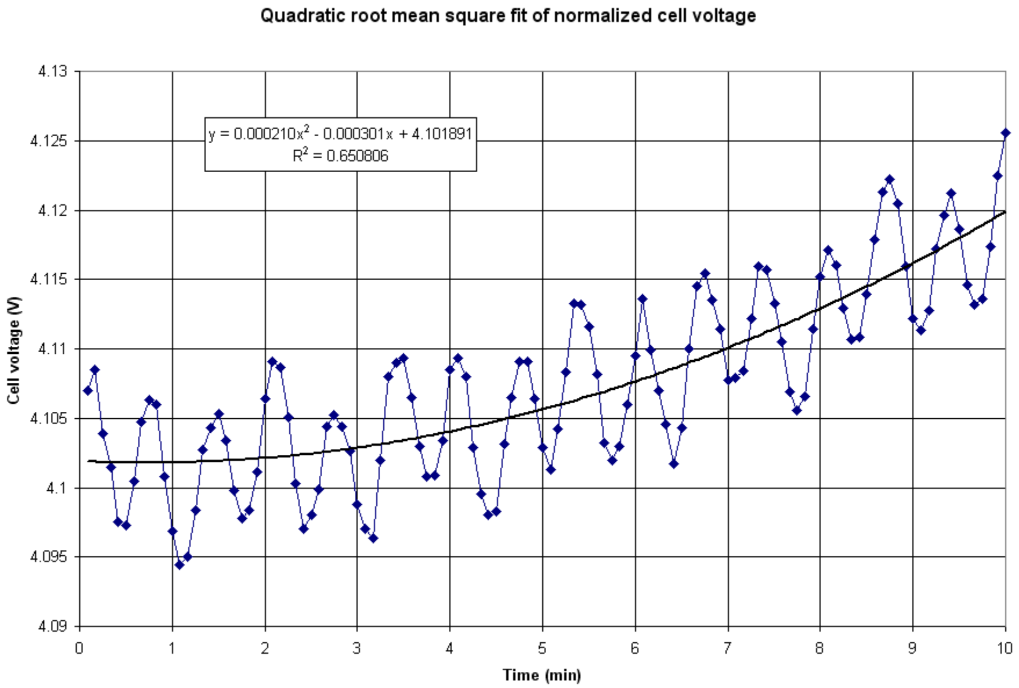


Figure 9: Quadratic root mean square fit of the normalized cell voltage

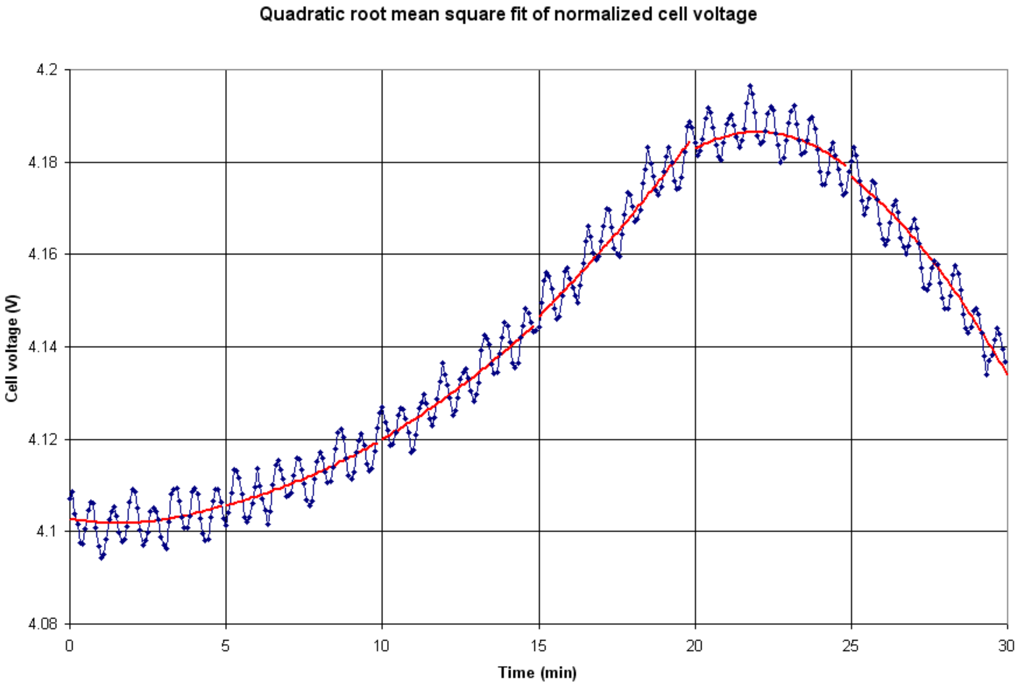


Figure 10: Quadratic root mean square fit of the normalized cell voltage

As for the linear fit, the quadratic or parabolic fit is discontinuous. Yet it is possible after the fact to take advantage of the overlap between the series of fitted curves to produce a single continuous curve by doing some interpolation. For example, the red curve in Figure 11 is produced by interpolating between the two overlapping fitted curves in the 15 to 20 minutes time segment. The interpolation is done is such a way that even the first derivative of the curve is continuous between time segments.

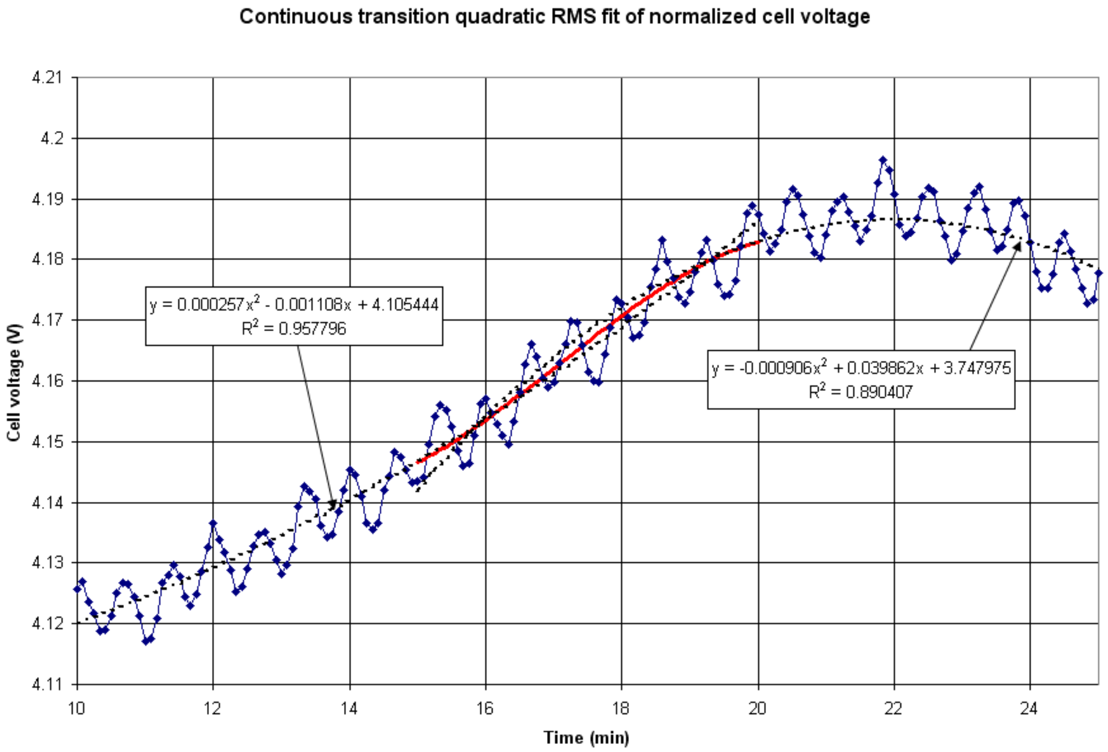


Figure 11: Continuous transition quadratic RMS fit of normalized cell voltage

Repeating this procedure from time segment to time segment produced the red curve presented in Figure 12. Evidently, this after-the-fact treatment is not helping improving the slope estimation at the end of the last available segment which is the current time when the information is required to take control actions as part of the cell control logic.

Yet, at least this red curve in Figure 12 can now be compared to the equivalent of the blue curve in Figure 4 to determine the appropriateness of the selected curve fitting algorithm.

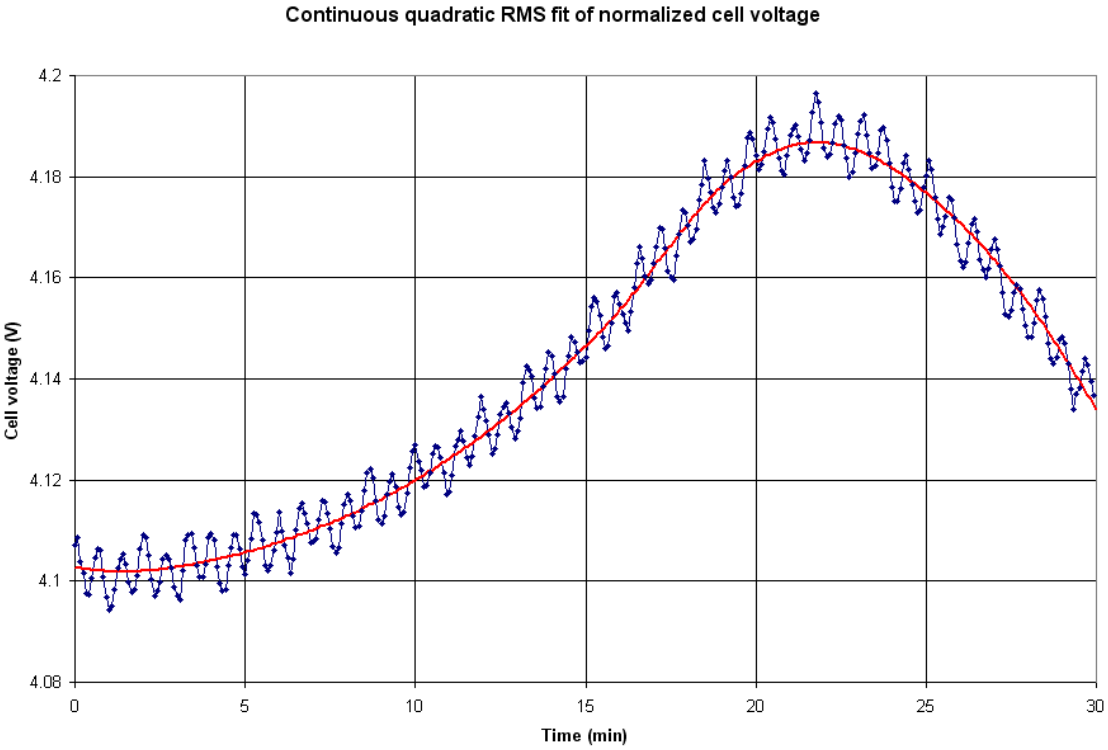


Figure 12: Resulting continuous quadratic RMS fit of normalized cell voltage

Estimation of the alumina concentration using the estimated slope of the noise free normalized cell voltage during a no feed observation period

The key concept of the *In Situ* control logic algorithm is the assumption that there is a reliably accurate correlation between the estimated slope of the noise free normalized cell voltage (labeled time slope of V_p in previous *In Situ* papers) and the alumina concentration during a no feed observation period (see per example Figure 1 of reference 3).

Tests performed on Dyna/Marc cell simulator indicates that this correlation does exist even if it is not linear on a very wide range of alumina concentration variation and depends on a lot of factors influencing the cell resistance and the dynamics of the alumina dissolution. For example, when the point fed alumina is dissolving rapidly, there is not much dispersed alumina in the bath at the beginning of a no feed observation. Under this condition, the rate of decrease of the dissolved alumina concentration in the bath reaches the constant rate of the cell alumina electrolytic consumption. Otherwise the rate of decrease of dissolved alumina is less than the maximal rate. Figure 13 presents the evolution of the dissolved alumina concentration during a 0% underfeeding followed by a 140% overfeeding continuous tracking feeding regime for 5 cycles. Figure 14 presents the corresponding evolution of the noise free cell voltage while Figure 15 presents the correlation between the slope of the cell voltage and the concentration of dissolved alumina for all 5 cycles.

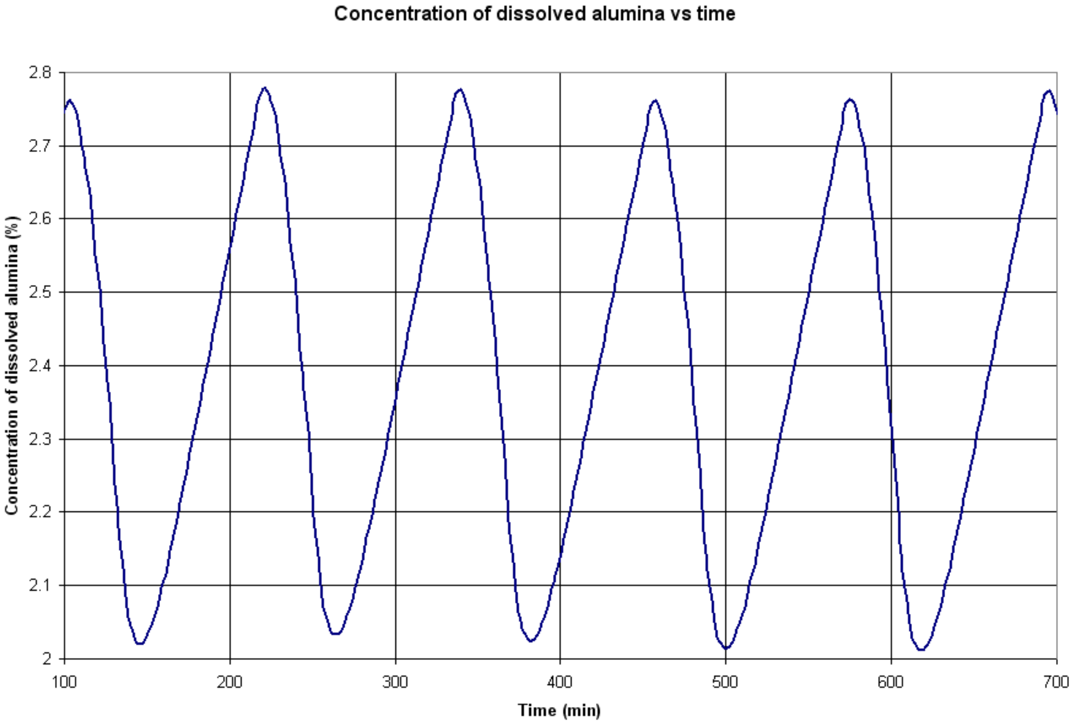


Figure 13: Concentration of dissolved alumina vs. time during continuous tracking

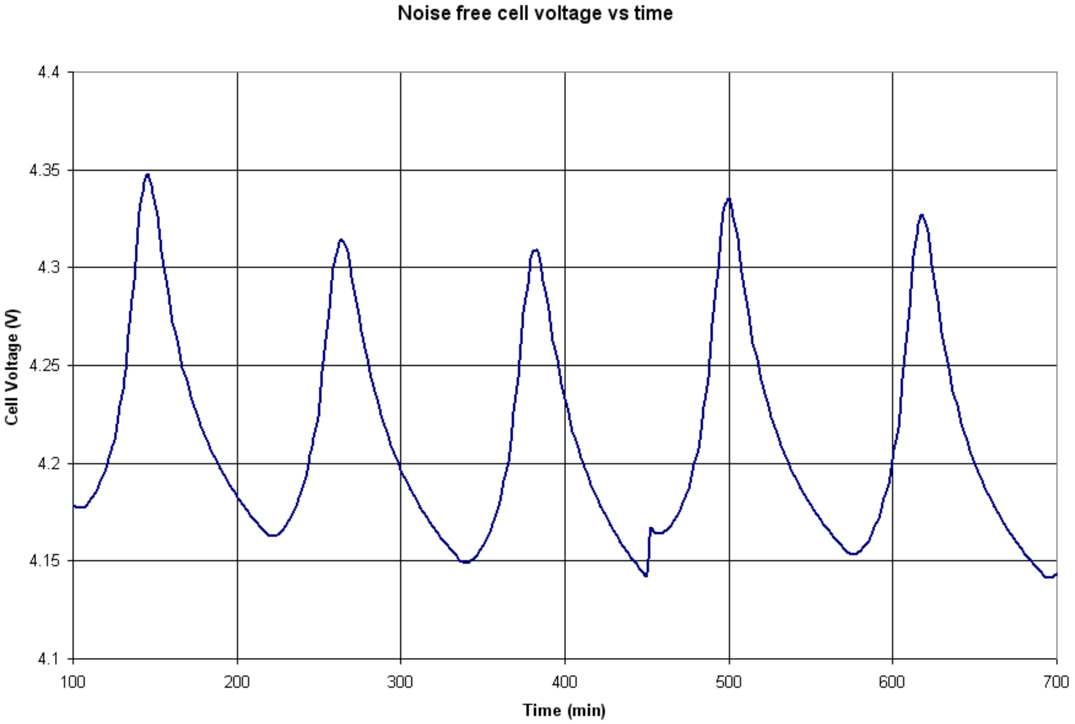


Figure 14: Noise free normalized cell voltage vs. time during continuous tracking

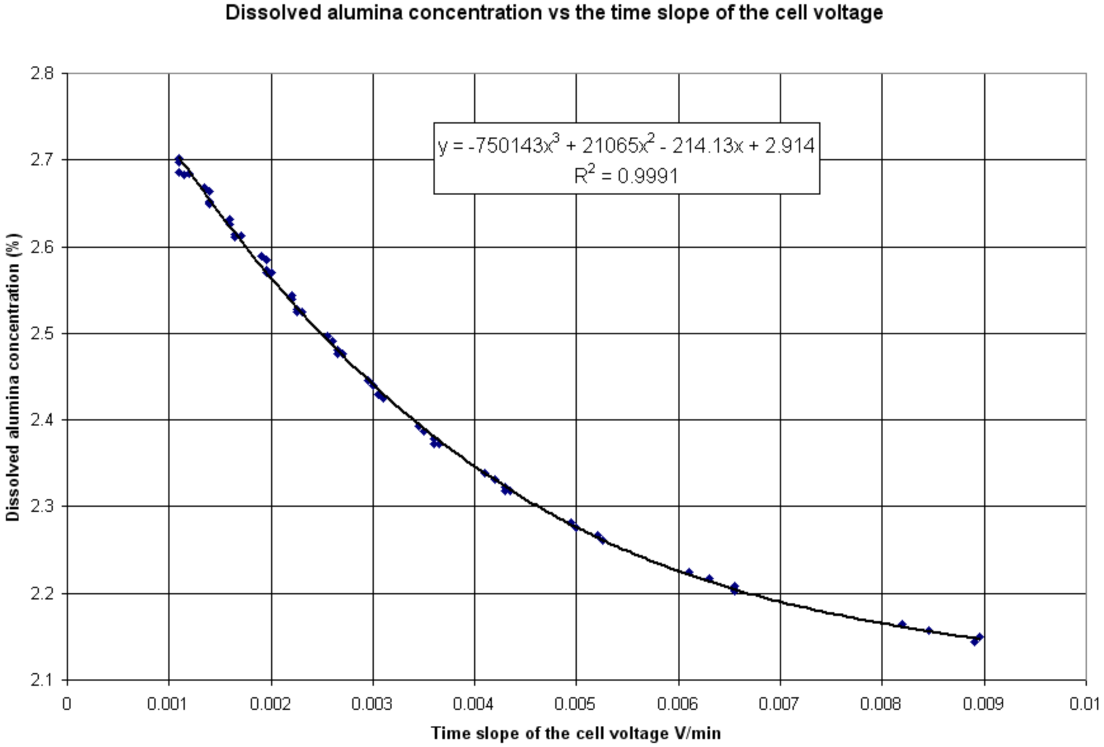


Figure 15: Dissolved alumina concentration vs. the slope of the cell voltage

So clearly even if the 5 cycles are not identical, they almost perfectly match a unique cubic correlation. For a more restricted range of alumina concentration variation which corresponds to the typical range the *In Situ* controller would be able to operate the cell and for a different value of the alumina dissolution constant, the linear correlation presented in Figure 16 was obtained. This is the correlation that will be used to estimate the alumina concentration once the slope of the noise free normalized cell voltage have been estimated by the *In Situ* controller in Dyna/Marc test runs.

For example, the slope of 3.9 mV/min estimated after 10 minutes of no feed observation by the quadratic fit of Figure 9 would correspond to an estimated alumina concentration of:

$$-0.0279 * 3.9 + 2.3193 = 2.21 \%$$

Using the correlation of Figure 15 would lead to a different estimate:

$$-750143 * 0.0039^3 + 21065 * 0.0039^2 - 214.13 * 0.0039 + 2.914 = 2.35 \%$$

This 0.14 % discrepancy between the real alumina concentration and the estimated alumina concentration (assuming that Figure 16 correlation gives an exact prediction) would not prevent the *In Situ* control logic to work quite well, it would simply introduce a 0.14% offset on the targeted alumina concentration.

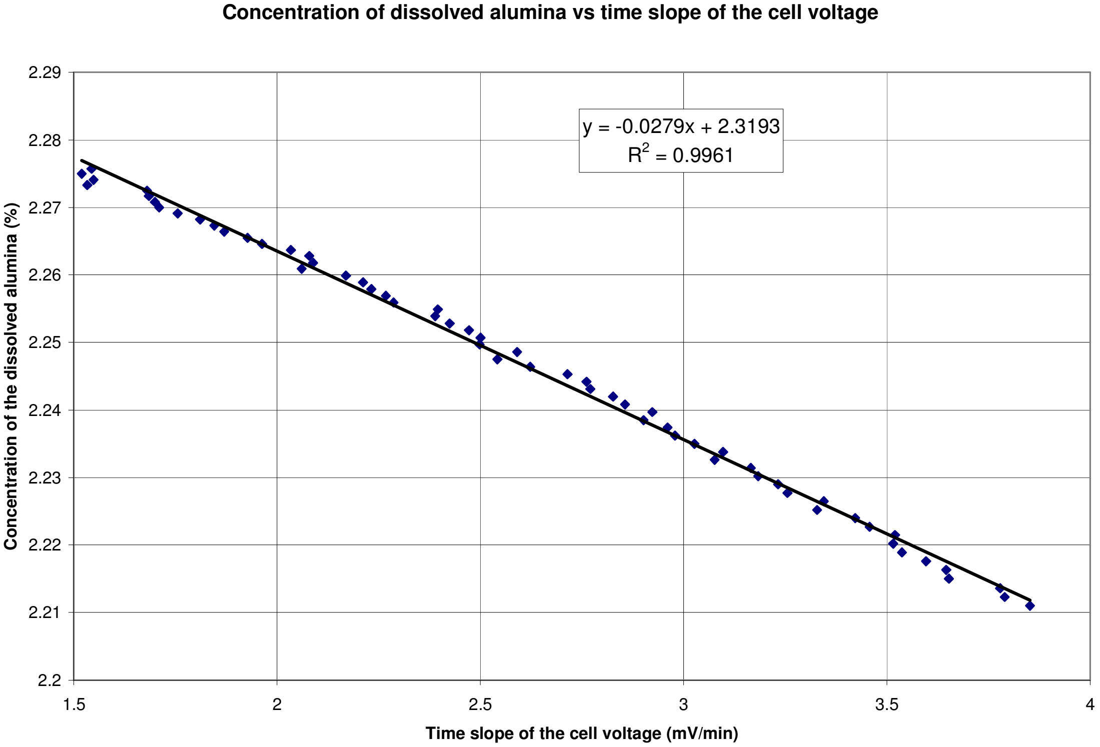


Figure 16: Dissolved alumina concentration vs. the slope of the normalized cell voltage

Primary calibration surface and estimation of the ACD

Once the concentration of dissolved alumina in the bath has been estimated, it is essential to also estimate the ACD using the primary calibration surface.

In previous *In Situ* papers, the concept of primary calibration curve is presented. It is simply the observation that when all the other variables affecting the cell voltage are kept constant, it is possible to correlate the concentration of dissolved alumina in the bath with the noise free normalized cell voltage using the following equation:

$$1 / (C_{Al2O3} - C_{Al2O3\text{-}AE}) = CoeffA * EXP (CoeffB * V_n) \qquad (2)$$

Where:

V_n	is the estimated noise free normalized cell voltage (V)
C_{Al2O3}	is the *In Situ* estimated concentration of dissolved alumina in the bath (%)
$C_{Al2O3\text{-}AE}$	is the estimated concentration of dissolved alumina triggering an anode effect (AE) (%)
CoeffA	is the first coefficient of the correlation
CoeffB	is the second coefficient of the correlation

It is quite easy to generate that correlation for any combination of cell conditions using published cell voltage break down equations like those published in [9] that happen to be the ones used by Dyna/Marc. This can be done in an Excel worksheet by specifying the value of all those other variables like the ACD, the cell temperature, the current density, anode cathode and busbar voltage drops, the bath chemistry etc., and calculating the resulting predicted cell voltage covering a range of values for the concentration of dissolved alumina. The exponential curve on the left of Figure 17 was obtained using 4 cm ACD while the right curve was obtained using 5 cm ACD.

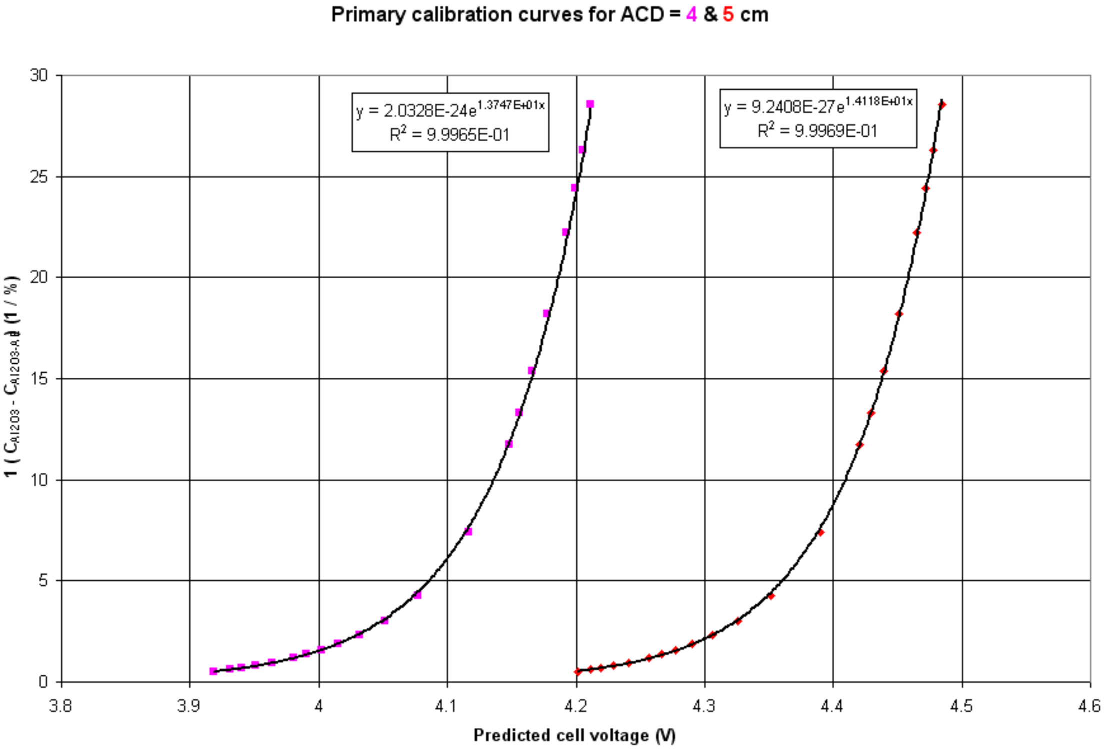


Figure 17: Primary calibration curves for ACD = 4 & 5 cm

For the cell conditions selected, CoeffA = 2.0328E-24 and CoeffB = 13.747 when ACD = 4 cm and CoeffA = 9.2408E-27 and CoeffB = 14.118 when ACD = 5 cm. With C_{Al2O3} = 2.210 % and $C_{Al2O3-AE}$ = 1.965 % we have:

1 / (CAl2O3 - CAl2O3-AE) = 1 / 0.245 = 4.08

From that we can calculate that at 2.21 % of dissolved alumina, the predicted cell voltage would be:

(LN(4.08) - LN(2.0328E-24)) / 13.747 = 4.071 V, at 4 cm ACD

(LN(4.08) - LN(9.2408E-27)) / 14.118 = 4.346 V, at 5 cm ACD

Since the estimated noise free normalized cell voltage after 10 minutes of no feed observation was estimated to be 4.1199 V from the quadratic fit presented in Figure 9, we can calculate by a simple interpolation method the estimated ACD:

4 + (4.1199 – 4.071) / (4.346 – 4.07) = 4.18 cm

Using the combined estimated cell voltage and slope of the cell voltage after 10 minutes of no feed observation, it is possible to estimate both the concentration of dissolved alumina in the bath and the ACD. From that point on, until the next observation, it is possible to estimate the evolution of the dissolved alumina concentration by simply following the evolution of the estimated noise free normalized cell voltage. In order to be able to do that, two things are needed:

1) An estimate of the evolution of the ACD

2) The primary calibration surface

By neglecting any changes of the metal level coming from changes of the ledge thickness or the accumulation of sludge, the ACD evolution depends only on the linear accumulation of the metal and the linear consumption of the anodes. For a prebaked anode cell, the net result is a slow linear decrease of the ACD.

In the demo Dyna/Marc test case, that linear rate of change of the ACD was estimated to be about -0.00033 cm/min, which means that 3 hours after the observation, the ACD can be estimated to be:

4.18 – 0.00033 * 180 = 4.12 cm

Of course, this prediction cannot be considered sufficiently accurate over a very long period of time of many hours. Yet, it only needs to be approximate until the next observation when ore feed is once again restricted for an *in situ* alumina concentration and ACD measurement.

Assuming that we can estimate the noise free normalized cell voltage every 5 minutes and provide an estimate of the ACD based on an assumed linear rate of change, we need to replace the primary calibration curve by a primary calibration surface. Contrary to the calibration curve that can only be used to estimate the concentration of dissolved alumina in the bath from the cell voltage at a given ACD, the primary calibration surface can be used to estimate the concentration of dissolved alumina in the bath from any combination of cell voltage and ACD.

It is possible to fit the evolution of the CoeffA and CoeffB coefficients as the ACD is changing from 4 to 5 cm as seen in Figures 18 and 19. By using those two extra equations, it is possible to calculate first the value of CoeffA and CoeffB that corresponds to the current estimate of the ACD and then calculate the estimate of the dissolved alumina in the bath based on the estimate of the noise free normalized cell voltage using those coefficients in Equation 2.

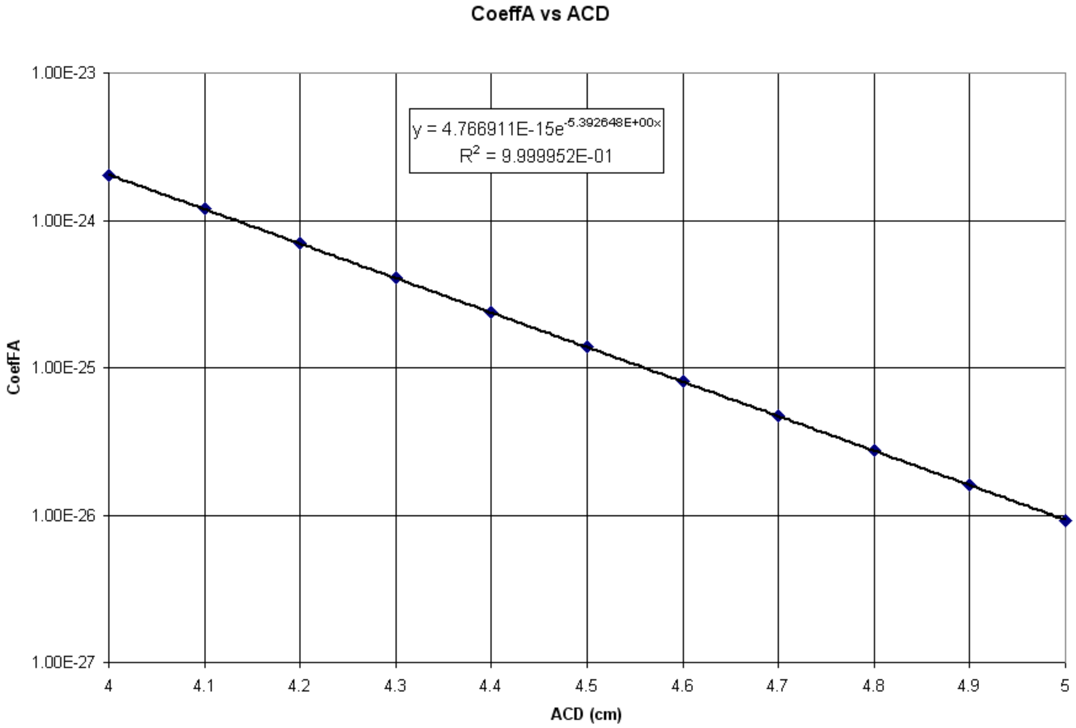


Figure 18: CoeffA vs. ACD

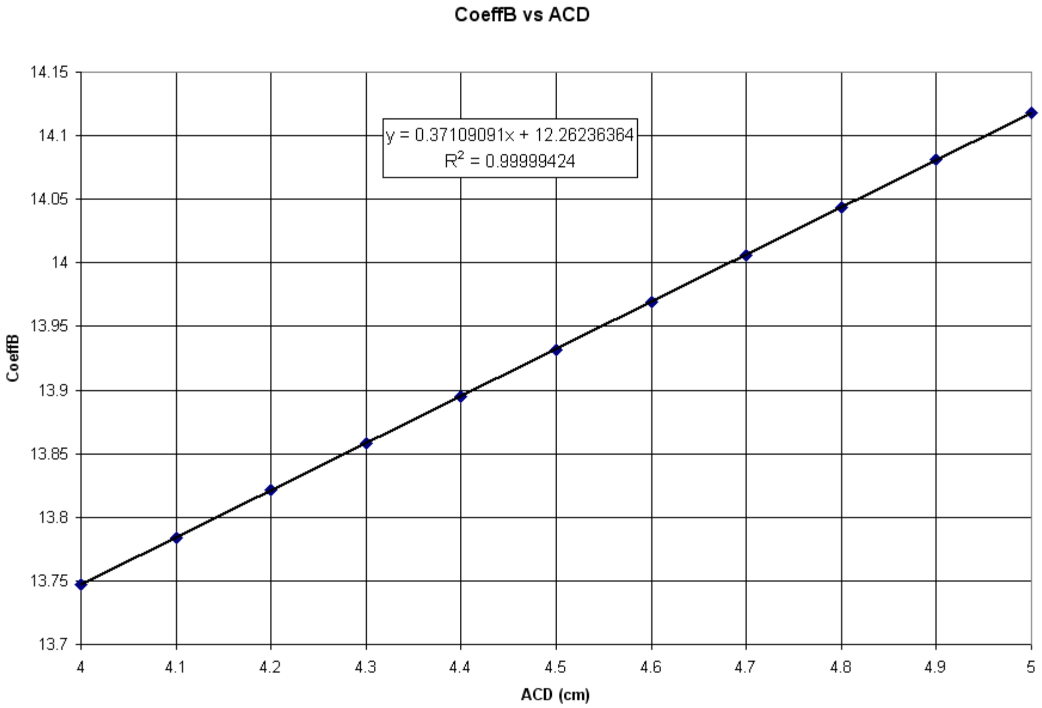


Figure 19: CoeffB vs. ACD

For example, using ACD = 4.12, CoeffA value can be calculated to be equal to:

4.766911E-15 * EXP (-5.392648 * 4.12) = 1.06957E-24

And CoeffB value can be calculated to be equal to:

0.37109091 * 4.12 + 12.26236364 = 13.7913

If 3 hours after the last observation, the value of the noise free normalized cell voltage is estimated to be equal to 4.100 V, the concentration of the dissolved alumina can in turn be estimated to be equal to:

1.06957E-24* EXP (13.7913 * 4.100) = 3.8557

1 / 3.8557 + 1.965 = 2.22 %

PID controller

The last part of the *In Situ* control logic is straightforward. After a no feed observation period, the concentration of the dissolved alumina in the bath is estimated every 5 minutes or so. Next, the error between a selected target value and the current estimated value is used to adjust the alumina feeding rate using the standard PID controller equation [10, 11]:

$$\mathrm{Output(t)} = K_p \left(e(t) + K_{ip} \int_0^t e(\tau)\, d\tau + K_{dp} \frac{de}{dt} \right) \qquad (3)$$

For example, starting with no integral and derivative contributions at the end of the observation period, a K_p value of 2000 kg/min% and a target alumina concentration of 2.30 % would translate into an additional feeding rate of:

2000 * (2.30 – 2.21) = 180 kg/hr

Since the nominal feeding rate of the demo Dyna/Marc test case is about 180 kg/hr, this means that the initial total feeding rate at the end of the observation period would be about 200% of the nominal rate (180 + 180). That feeding rate would be maintained for 5 minutes and then readjusted based on the new estimated value of the dissolved alumina concentration at the time.

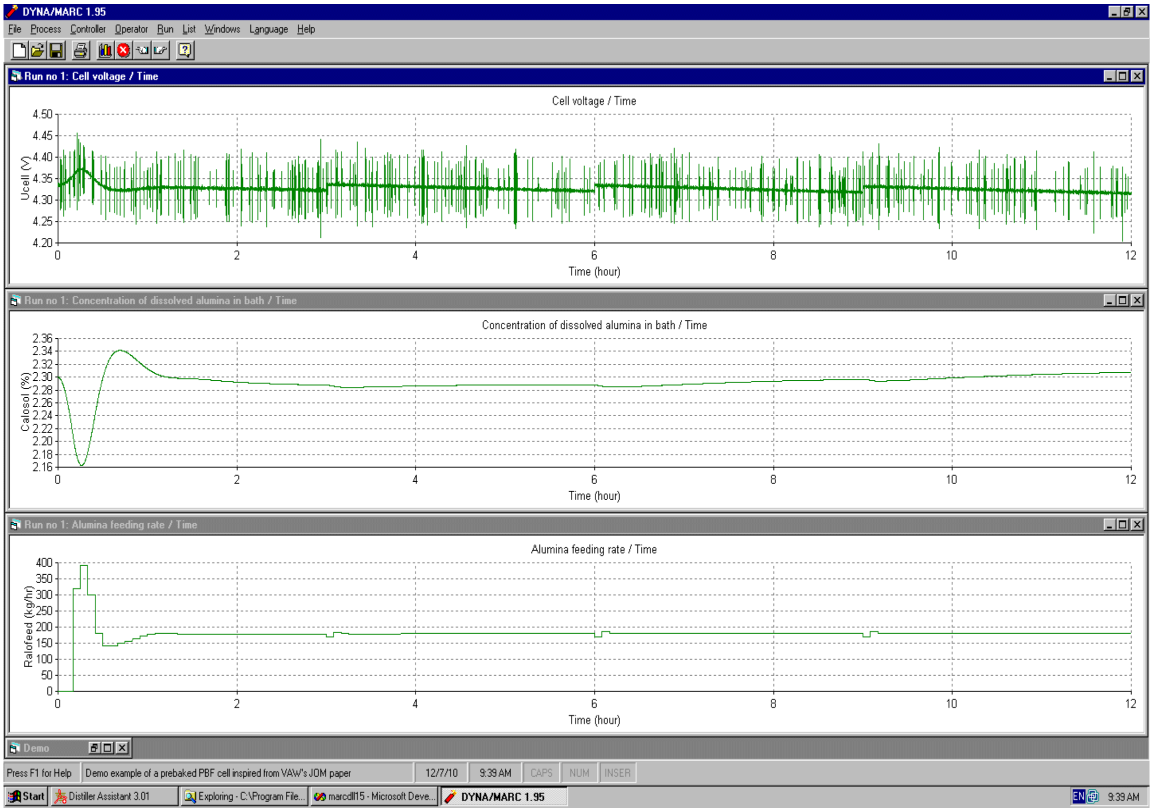
This summarizes the theory behind the *In Situ* control logic. It is clearly an example of “out of the box” thinking as nothing even remotely similar was ever imagined to control the alumina concentration of a H.H. cell. Would this innovative control logic work in a real cell? It is difficult to say with absolute certainty at this stage as a number of assumptions are built into the logic. Continued testing in the Dyna/Marc cell simulator will provide these answers in a risk free environment! This is one of the reasons why cell simulators have been developed in the first place.

TESTING *IN SITU* CONTROL WITH DYNA/MARC CELL SIMULATOR

For the purpose of the Dyna/Marc test, the simplest version of the *In Situ* control logic has been coded in Fortran, the language in which Dyna/Marc kernel was developed. In the simplest version, only the linear curve fitting option is available.

The simplest version of the *In Situ* controller subroutine has been added to Dyna/Marc and tested. In the test runs, the CO_2 gas release noise generation is neglected so the selected data sampling time step is 5 seconds. The *In Situ* calculation step is 5 minutes which produces 60 normalized cell voltage data points. In normal operation after the observation period, the estimation of the noise free normalized cell voltage is used to calculate the estimate of the concentration of dissolved alumina, the PID controller error and finally the new feeding regime.

The Dyna/Marc simulator runs start in perfect steady-state condition as any Dyna/Marc run does at the target ACD and target alumina concentration of 2.3%. A no feed observation period is started immediately. As can be seen in Figure 20, the perturbation created by the observation last about an hour, after which the cell is back in perfect steady-state conditions. The ACD is adjusted every 3 hours which create very little perturbations on the feeding rate.



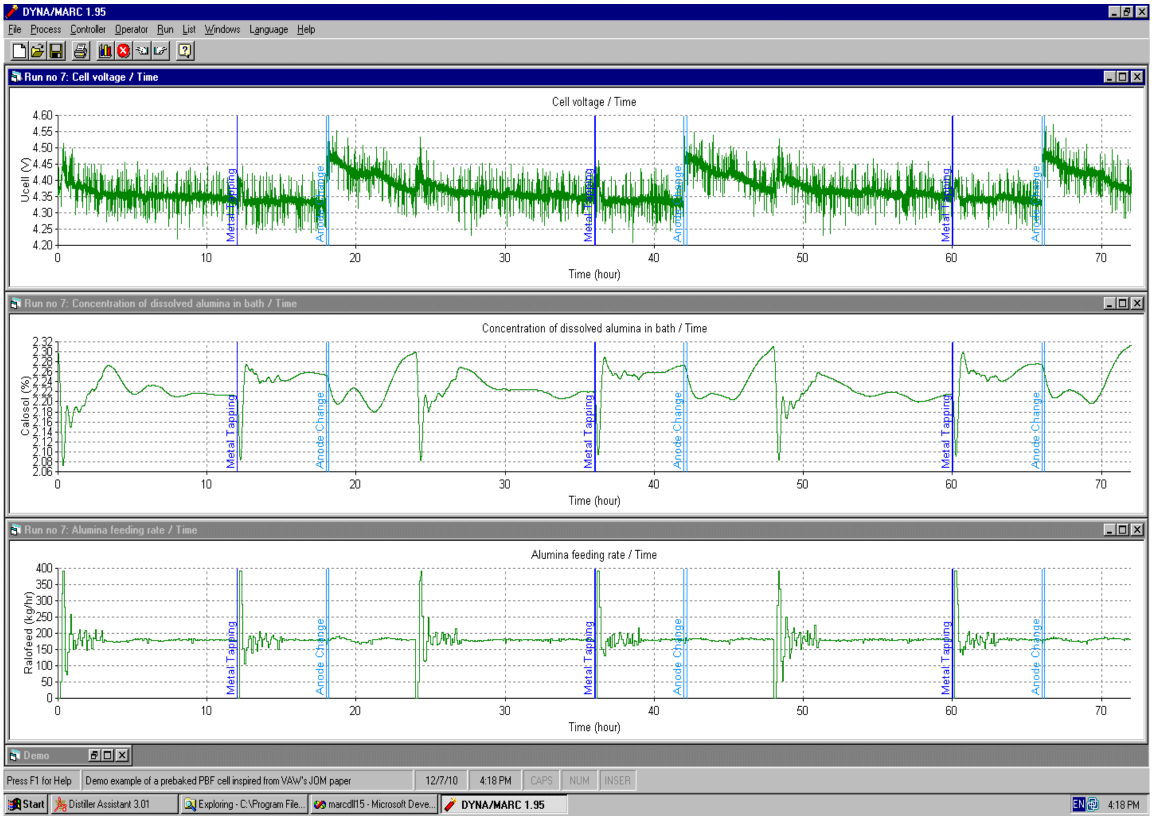
Demo example of a prebaked PBF cell inspired from VAW's JOM paper

Figure 20: 12 hours Dyna/Marc simulation using *In Situ* control logic

This clearly demonstrates that without major process perturbations, *In Situ* control logic works exactly as designed. The dissolved alumina concentration is effectively controlled with an impressive accuracy employing only standard line amperage and cell voltage signals as feedback control inputs.

Yet, a 12 hours simulation period without process events like metal taping or anode change is not enough to demonstrate that the logic is sufficiently robust to cope with typical unavoidable process perturbations. For sure, the anode change event affects the cell resistance and this effect must be accounted for in the *In Situ* logic. Otherwise *In Situ* logic will assume the change is generated by a change of the dissolved alumina concentration and it would very inappropriately change the feeding regime accordingly.

It turns out that it is possible to account for the effect of the anode change on the cell resistance by using a ramp the same way it is done in a standard cell controller. Figure 21 presents the results of a 3 days simulation containing 3 metal tapping events and 3 anode change events. The anode change event does perturb significantly the *In Situ* control logic despite the best possible compensation of the resistance change with a ramp. Yet, when compared with a standard continuous tracking control logic, those perturbations remain relatively very small.



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Demo example of a prebaked PBF cell inspired from VAW's JOM paper

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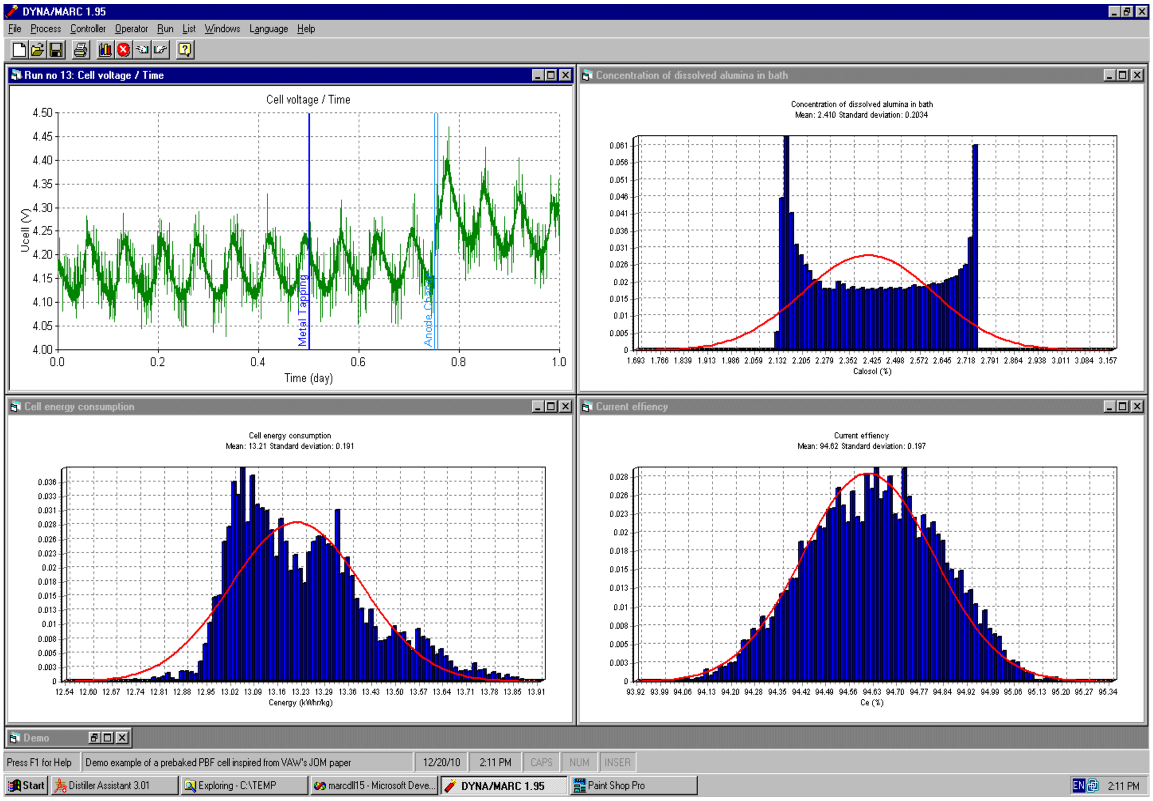
Figure 21: 3 days Dyna/Marc simulation using the *In Situ* control logic

Effect of the change of control logic on the process efficiency

As a last test, the average energy efficiency and current efficiency of a 1 day run was compared when only the type of feed control logic is changed. Figure 22 presents the results obtained using a typical continuous tracking control logic. The average dissolved alumina concentration obtained is 2.41 % with a standard deviation of 0.20 %, the average power efficiency is 13.21 kWhr/kg with a standard deviation of 0.19 kWhr/kg and an average current efficiency obtained is 94.62 % with a standard deviation of 0.20 %.



Demo example of a prebaked PBF cell inspired from VAW's JOM paper

Figure 22: Process efficiency results during a typical continuous tracking logic

Figure 23 presents the results obtained using the new *In Situ* control logic. The average dissolved alumina concentration obtained is 2.22 % with a standard deviation of only 0.03 %, the average power efficiency is 13.16 kWhr/kg with a standard deviation of 0.12 kWhr/kg and an average current efficiency obtained is 94.77 % with a standard deviation of 0.12 %. Further *In Situ* optimizing efforts are expected to produce a wider divergence in production efficiencies.

Therefore the drastic reduction of the process variation of the concentration of the dissolved alumina leads to the possibility of reducing the average value without increasing the risk of anode effect. This result had a positive impact on both the current efficiency and the power efficiency.

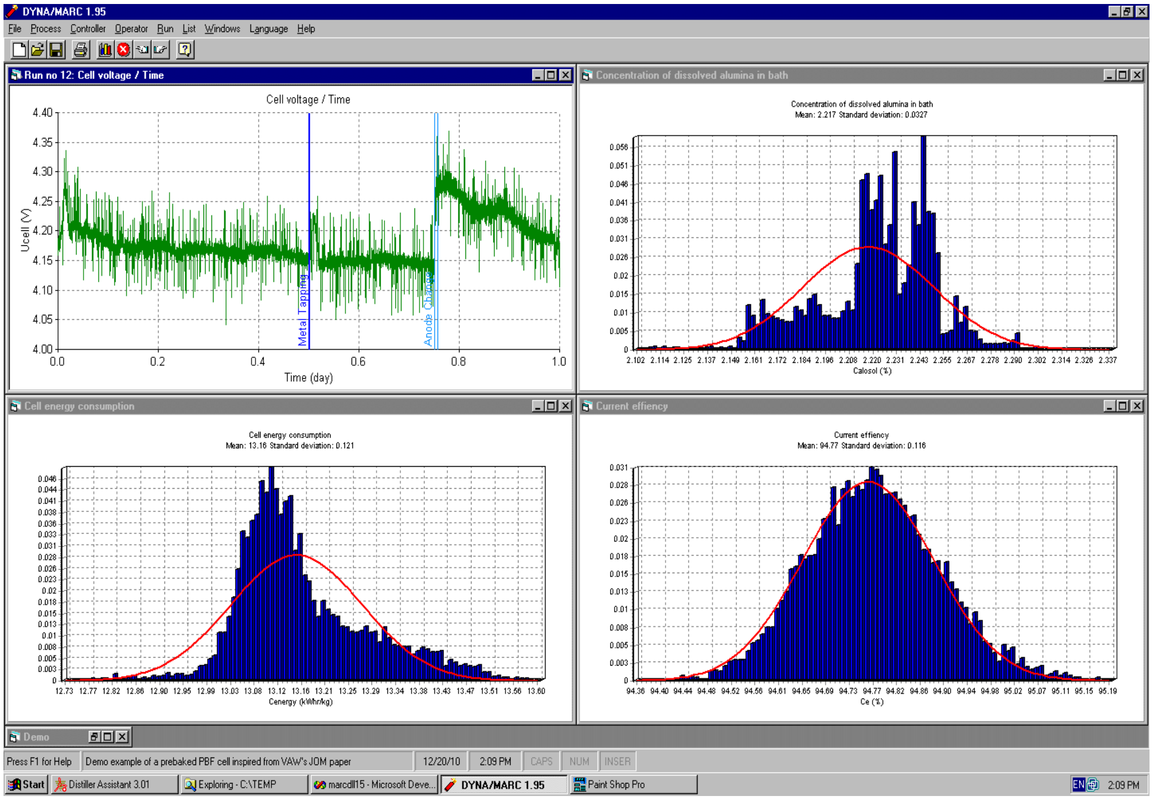


Figure 23: Process efficiency results employing the new *In Situ* control logic

CONCLUSIONS

The newly proposed *In Situ* control logic has been presented in detail. The core principle is quite simple: the values of both the dissolved alumina concentration and the ACD can be estimated at the end of a 5 to 10 minute no feed observation period only by using the standard cell amperage and voltage data. With this information, it is possible to employ a PID controller to regulate the rate of alumina feeding using only the normalized cell voltage to calculate the error between the estimated dissolved alumina concentration and a target value for an extended period of time.

In Situ control logic has been coded in Fortran and imbedded in the Dyna/Marc kernel as an additional control subroutine and tested. It has worked exactly as designed. The dissolved alumina concentration is effectively controlled with an impressive accuracy only using standard line amperage and cell voltage signals as feedback control inputs. The drastic reduction of the process variation of the concentration of the dissolved alumina, promotes the possibility of reducing the averaged value without increasing the risk of anode effect. This new idea produced in the Dyna/Marc simulator a positive impact on both current efficiency and the power efficiency.

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