Presented at 20th Internat.Congress Theor.Appl.Math., Chicago 27August 2000.

Non-linear wave models for aluminium reduction cells

V.Bojarevics[1], K.Pericleous[1] and J.Freibergs[2]
[1] *University of Greenwich, CMS, 30 Park Row, SE10 9LS, London, UK*
[2] *Institute of Physics, Salaspils-1, Latvia*

An industrial electrolysis cell used to produce aluminium can sustain highly coupled non-linear waves at the interface of liquid aluminium and electrolyte. The interface waves are similar to stratified sea layers, but the penetrating electric current and the associated magnetic field are intricately involved in the oscillation process, and the observed wave frequencies are shifted from the purely hydrodynamic ones. The wave development depends on the magnetic field distribution created by the current supplying busbars and the current within the cell. The interface stability problem is of great practical importance because the electrolytic aluminium production is a major electrical energy consumer, and it is related to environmental pollution rate.

In Ref. [2] a systematic perturbation expansion is developed for the fluid dynamics and electric current problems which permitted to reduce the three-dimensional problem to a two-dimensional one for the leading order expansion terms. The procedure is more generally known as "shallow water approximation" which can be extended for the case of weakly non-linear and dispersive waves. The Boussinesq formulation permits to generalise problem for non-unidirectionally propagating waves, accounting for side walls and for a two fluid layer interface. E.g., Renouard et al. [4] found a good correspondence to the experimental results for resonantly interacting waves in a channel mounted on a rotating platform.

In [1] shallow layer generalised Boussinesq equations were derived for wave motions depending on two horizontal variables x,y for two layers and with the electromagnetic interaction which makes the flow velocities rotational. Thus the velocity potential alone will not be adequate for the flow representation. The depth average horizontal velocity $\mathbf{u}_0(t,x,y)$ can be decomposed with the following properties:

$$\begin{aligned} u_{xo} &= -\partial_y \psi + \partial_x \chi \\ u_{yo} &= \partial_x \psi + \partial_y \chi \end{aligned} \; ; \qquad \begin{aligned} (curl\ \mathbf{u}_o)_z &= \partial_{kk} \psi \\ div_2 \mathbf{u}_o &= \partial_{kk} \chi \end{aligned} \; ; \qquad k = 1, 2 , \tag{1}$$

where the velocity potential $\chi(t,x,y)$ is determined essentially by the interface $z = \varsigma(t,x,y)$ variation, and the stream function $\psi(t,x,y)$ – by the rotational body force action. Moreover, these functions are intricately coupled owing to the problem non-linearity and, most importantly, by the electromagnetic field dependence on the interface ς variation. The depth average momentum and continuity equations for the two fluid layers can be combined in one non-linear wave equation for the interface:

$$\left\langle \frac{\rho}{h} \right\rangle \partial_{tt} \varsigma - \frac{1}{3} \langle \rho h \rangle \partial_{ttjj} \varsigma + \left\langle \frac{\mu \rho}{h} \right\rangle \partial_t \varsigma + \langle \rho g \rangle \partial_{jj} \varsigma = \langle \partial_j F_j \rangle - \left\langle \frac{\rho}{h} \partial_{tj} (\varsigma u_{jo}) + \rho \partial_j (u_{ko} \partial_k u_{jo}) + \frac{\mu \rho}{h} \partial_j (\varsigma u_{jo}) \right\rangle \tag{2}$$

where $\langle F \rangle = F_1 - F_2$ denotes a difference of variables in the two layers, F_j – the electromagnetic force components (effectively including the vertical force), ρ_i – density, μ_i – friction coefficient [1], h_i – undisturbed depth for each layer (h_1=0.25, h_2=0.05 m in the following computed examples).

We consider different levels of approximation to the problem, starting with the non-linear gravity wave in the absence of the electromagnetic force. If starting with (1,0) natural gravity wave perturbation, the difference between the linear and non-linear sloshing wave development is demonstrated in Fig.1. The final non-linear wave spatial profile is very close to the analytical solution by Tadjbakhsh & Keller [5]. With the three layer electric current model [2] and the given magnetic field, which is computed for a specific 122 kA industrial cell [3] including the full 3-dimensional bus-bar representation and the steel elements (Fig.2), the same initial perturbation grows over a large time period of 1000 s even with the friction included (Fig 3). The initial perturbation is completely damped and the magnetically shifted frequency is dominant. The model with the turbulent fluid flow as a part of the wave dynamics, with 3-dimensional electric current and magnetic field continuously updated, leads to a stable oscillation of the similar frequency (Figs. 3 and 4).

References.

[1] Bojarevics V., *Progr. Fluid Flow Res.: Turbulence and Applied MHD, eds. H.Branover and Y.Unger, AIAA, 1998, Chapter 58, pp.833-848.*
[2] Bojarevics V. and Romerio M.V., *Eur. J. Mech., B/Fluids*, Vol.13, No 1, 1994, pp.33-56.
[3] Freibergs J., *Proc. Int. Colloq. 'Modelling Material Processing' Riga, Latvian University*, 1999, pp.86-91.
[4] Renouard D.P., Tomasson G.G., Melville W.K., *Phys. Fluids, A*, Vol. 5, No 6, 1993, pp.1401-1411.
[5] Tadjbakhsh I. and Keller J.B., *J. Fluid Mech.*, Vol. 8, 1960, pp.442-451.

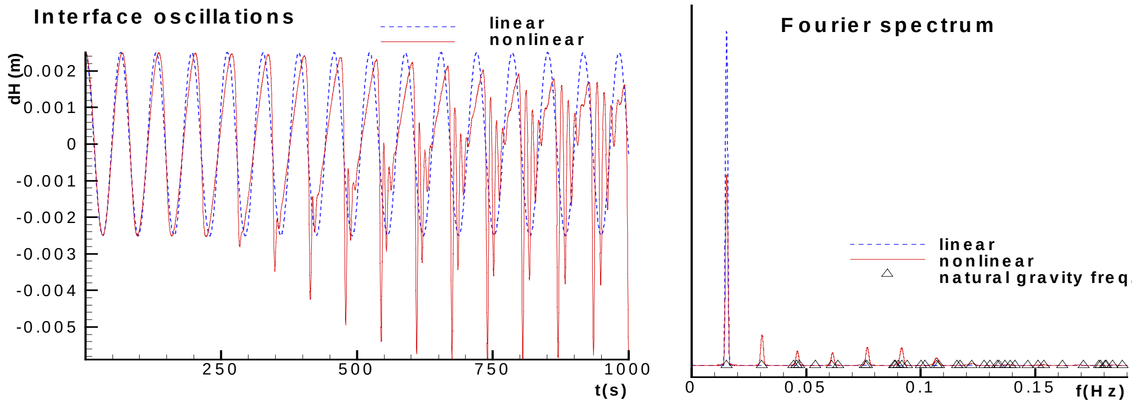

Figure 1. Linear and non-linear wave evolution from the (1,0) natural gravity wave initial perturbation.

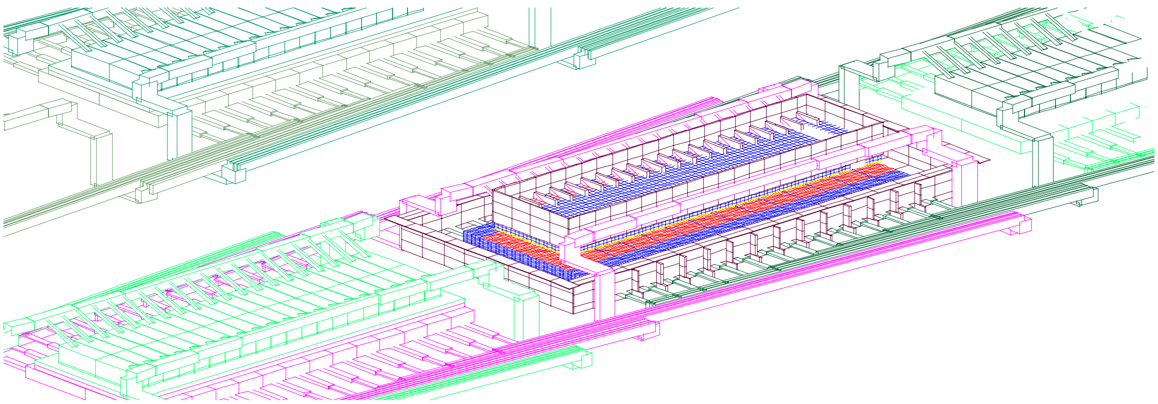

Figure 2. Soderberg horizontal stud reduction cell Autocad representation of the elements used for the magnetic field and electric current modelling.

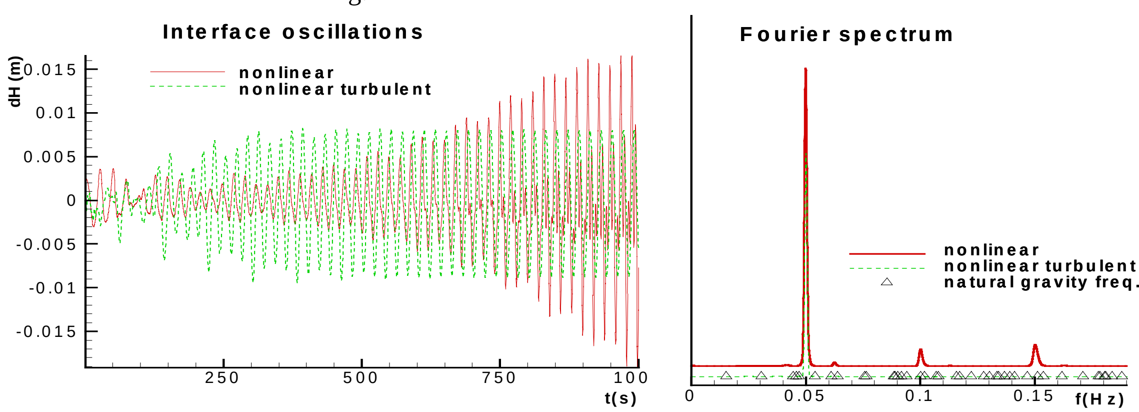

Figure 3. Non-linear wave evolution from the (1,0) perturbation with the effect from the magnetic field for the Soderberg cell.

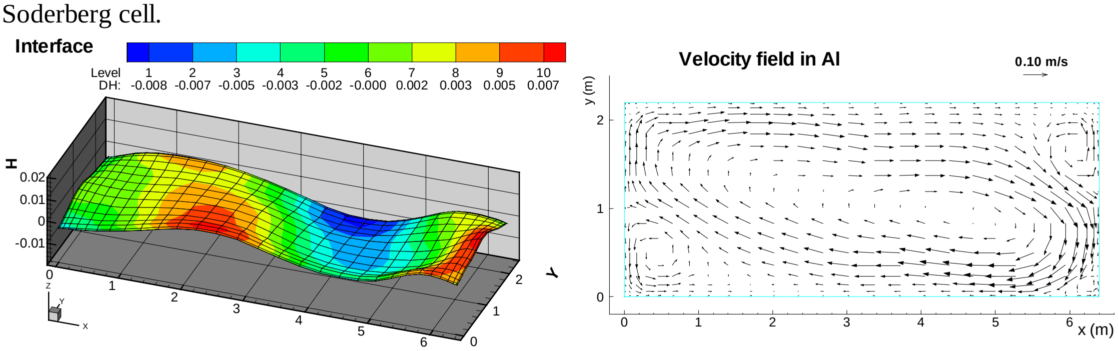

Figure 4. The instantaneous interface shape and the turbulent velocity field in the liquid metal at t = 1000s.